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Vector meson photoproduction from the BFKL equation 1. Theory

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Abstract: Diffractive vector meson photoproduction accompanied by proton dissociation is studied for large momentum transfer. The process is described by the non-forward BFKL equation, for which a complete analytical solution is found. The scattering amplitudes for all combinations of helicity are presented.

Keywords: LEP HERA and SLC Physics, QCD.

Contents

1. Introduction

Diffractive vector meson photoproduction at large momentum transfer, $\gamma p \to VX$, is an interesting process experimentally, because of the clean signal: an isolated vector meson with large transverse momentum, that is separated from the proton remnant by a large rapidity gap. Theoretically, it is advantageous because the large momentum transfer $|t|$ provides the hard scale necessary for a perturbative QCD description of the process in terms of hard colour singlet exchange. It has therefore been proposed as an ideal testing ground for BFKL dynamics [1, 2].

This process was recently investigated at HERA [3, 4]. The ZEUS analysis includes the angular distribution of the decay products, giving access to helicity amplitudes of the photon-meson transition. The measurements gave somewhat surprising results. Both for ρ and ϕ, the dependence of the cross-section on the momentum transfer t is approximately power-like, $|t|^{-n}$, with the exponent $n \simeq 3$. It can be deduced from the data that the leading contribution comes either from the process with no helicity flip or from the one with double helicity flip. The subleading helicity amplitudes were shown to be one order of magnitude smaller than the dominant one.

An analysis based on leading-order perturbative QCD predicts a steeper decrease of the cross-section, $n \simeq 4$, for the no-flip and double-flip amplitudes [5]. The single-flip amplitude M_{+0} gives a differential cross section with $n \simeq 3$, and should become dominant at very large momentum transfer $|t|$. It is clear that these properties are in contradiction with the experimental results.

Improved theoretical understanding came about after the suggestion that the real photon may couple to a quark-antiquark pair with a significant chiral odd component [5]. In general, the quark-antiquark pair can couple with either chiral even or chiral odd components, the latter vanishing in perturbation theory in the limit of massless quarks. However, it is not clear that the mass should be interpreted as the current quark mass. The answer to this question turns out to be essential for the phenomenology. Estimates based on QCD sum rules were performed in [5], giving a characteristic value of the non-perturbative mass parameter rather close to the constituent quark mass for light vector mesons. At moderate $|t| \sim 10\,\mathrm{GeV}^2$, the chiral odd amplitude with no helicity flip (M_{++}^{odd}) is then expected to dominate. In this paper we compute the chiral odd and chiral even amplitudes, leaving our results explicitly in terms of the quark mass m. An alternative explanation for the dominance of M_{++} and the emergence of dimensional scaling can be found in [6]. We shall return to discuss the connection of our approach with that of [6] in our second paper, where we shall focus on the phenomenology [7].

The analysis of [5] was restricted to the lowest order approximation for colour singlet exchange, that is the exchange of two gluons. In fact, the diffractive production of vector mesons occurs at energies $\sqrt{s}$ much larger than the momentum transfer. In such kinematics, the perturbative QCD corrections to two gluon exchange are enhanced by powers of large logarithms of the energy, $\log(s/|t|)$. The leading logarithmic corrections are proportional to $[\alpha_s \log(s/|t|)]^n$ at the nth order of the perturbative expansion. Thus, it is not sufficient to consider the lowest order approximation. A convenient way to perform the resummation of leading logarithmic terms to all orders in α_s is given by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [8]. This approach turned out to be successful in the case of diffractive production of charmonia at high $|t|$ [1, 2, 9, 10, 11]. Interestingly enough, the t-dependence of vector meson production was successfully described in [10] by a BFKL fit with a non-relativistic wave function for both light and heavy mesons, we shall comment further on this finding later.

The purpose of this paper is to present analytic results for the chiral-even and chiral-odd helicity amplitudes with complete leading logarithmic BFKL resummation. This calculation will give control of important QCD effects in vector meson photoproduction at high energies. A detailed comparison to available data will be performed in a forthcoming paper [7].

The paper is constructed as follows: The basic ideas are introduced in section 2, the helicity amplitudes at the lowest order are given in section 3 and their BFKL evolution is found in section 4. Brief conclusions can be found in section 5.

2. Hard colour singlet exchange

Let us consider a frame in which the photon-proton collision occurs along the z axis. The momentum transfer vector q is dominated by its transverse part $\boldsymbol{q}$, $t = q^2 \simeq -\boldsymbol{q}^2$. The possible helicity states of the incoming quasi-real photon are characterized by transverse polarization vectors

$$\boldsymbol{\epsilon}^{\pm} = \mp \frac{1}{\sqrt{2}}(1, \pm i)\,, \tag{2.1}$$

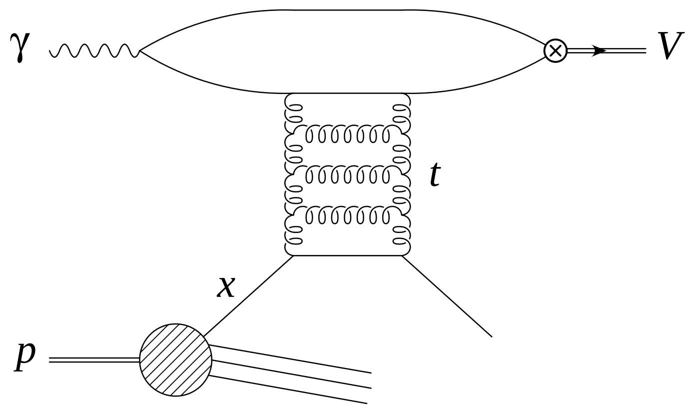

Figure 1: Vector meson photoproduction at large momentum transfer via hard colour singlet exchange.

and similarly for the transversely polarized vector mesons. For the meson, though, the longitudinal polarization ϵ^0 is also allowed. Thus, the possible photon-meson transitions may be described using three independent helicity amplitudes M_{++} (no-flip), M_{+0} (single-flip) and M_{+-} (double-flip).[1]

The diffractive process $\gamma p \to VX$ at large momentum transfer t (see figure 1) takes place by exchange of the BFKL pomeron. It has been demonstrated that at large momentum transfer, the hard pomeron couples predominantly to individual partons in the proton [12]. Thus, the cross-section may be factorized into a product of the parton level cross-section and the parton distribution functions,

$$\frac{d\sigma(\gamma p \to VX)}{dt\,dx_j} = \left(\frac{4N_c^4}{(N_c^2-1)^2}G(x_j,t) + \sum_f [q_f(x_j,t) + \bar{q}_f(x_j,t)]\right)\frac{d\sigma(\gamma q \to Vq)}{dt}, \quad (2.2)$$

where $N_c = 3$, $G(x_j,t)$ and $q_f(x_j,t)$ are the gluon and quark distribution functions respectively, and W^2 is the γp centre-of-mass energy squared. The struck parton in the proton, that initiates a jet in the proton hemisphere, carries the fraction x_j of the longitudinal momentum of the incoming proton. For future use, we introduce the integrated quantity

$$\left\langle\frac{d\sigma(\gamma q \to Vq)}{dt}\right\rangle = \int_{\rm cuts} dx_j\,\frac{d\sigma(\gamma p \to VX)}{dt\,dx_j}\,. \quad (2.3)$$

The partonic cross-section, characterized by the invariant collision energy squared $\hat{s} = x_j W^2$ is expressed in terms of the amplitudes $M_{\lambda\lambda'}(\hat{s},t)$,

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\left(|M_{++}(\hat{s},t)|^2 + |M_{+0}(\hat{s},t)|^2 + |M_{+-}(\hat{s},t)|^2\right). \quad (2.4)$$

In the case of an unpolarized lepton (photon) beam, it is impossible to measure directly the helicity amplitudes due to interference effects. Thus, in the experiment one

[1] There are also the corresponding amplitudes M_{--}, M_{-0} and M_{-+} which satisfy $M_{++} = M_{--}$, $M_{+-} = M_{-+}$ and $M_{+0} = -M_{-0}$.

selects a sample of vector mesons and measures the angular decay distribution of the decay products [3]. The following spin density matrix elements can be determined:

$$r_{00}^{04} = \frac{\langle |M_{+0}|^2 \rangle}{\langle |M_{++}|^2 + |M_{+0}|^2 + |M_{+-}|^2 \rangle}\,, \tag{2.5}$$

$$r_{10}^{04} = \frac{1}{2}\frac{\langle M_{++}M_{+0}^{*} + M_{+-}M_{-0}^{*} \rangle}{\langle |M_{++}|^2 + |M_{+0}|^2 + |M_{+-}|^2 \rangle}\,, \tag{2.6}$$

$$r_{-11}^{04} = \frac{\langle \mathcal{R}\mathrm{e}\,(M_{++}M_{+-}^{*}) \rangle}{\langle |M_{++}|^2 + |M_{+0}|^2 + |M_{+-}|^2 \rangle}\,, \tag{2.7}$$

where $\langle \cdots \rangle$ denotes the integration of the parton level quantities over partonic x_j with the appropriate cuts (c.f. (2.3)).

3. The lowest order amplitude

In the high energy limit it is convenient to use the dipole representation for the calculation of the scattering amplitude. In this limit the helicities of quarks, the large light-cone components of their momenta, and the transverse positions, are conserved in the scattering process. The small components of light-cone momenta are integrated out. Thus, the impact factor corresponding to a transition of a photon with helicity λ to a meson with helicity λ', due to the coupling of two gluons with momenta k and $q-k$, may be represented in the following way

$$\Phi^{\gamma(\lambda)V(\lambda')}(\boldsymbol{k},\boldsymbol{q}) = \int du \int d^2\boldsymbol{r}\, [\Psi^{*}_{V(\lambda')}(u,\boldsymbol{r})]_{\alpha\beta}\, T(\boldsymbol{k},\boldsymbol{q};\boldsymbol{r},u)\, [\Psi_{\gamma(\lambda)}(u,\boldsymbol{r})]_{\alpha\beta}\,, \tag{3.1}$$

where Ψ_V (Ψ_γ) denotes the vector meson (photon) wave function, u ($\bar{u} = 1-u$) is the quark (antiquark) longitudinal momentum fraction, and $\boldsymbol{r}$ the transverse quark-antiquark separation vector. Summation over spinor indices α and β is performed. $T(\boldsymbol{k},\boldsymbol{q};\boldsymbol{r},u)$ is the hard amplitude of the QCD dipole scattering, which, up to a constant factor, equals

$$f^{\mathrm{dipole}} = e^{i\boldsymbol{q}\boldsymbol{r}u}\left(1 - e^{-i\boldsymbol{k}\boldsymbol{r}}\right)\left(1 - e^{-i(\boldsymbol{q}-\boldsymbol{k})\boldsymbol{r}}\right), \tag{3.2}$$

see [5]. If we consider the meson photoproduction off a quark, the complete amplitude takes the form:

$$M_{\lambda\lambda'}(q) = \int d^2\boldsymbol{k}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\Phi^{\gamma(\lambda)V(\lambda')}(\boldsymbol{k},\boldsymbol{q})\,\Phi^{qq}(\boldsymbol{k},\boldsymbol{q})\,, \tag{3.3}$$

where the quark impact factor $\Phi^{qq}(\boldsymbol{k},\boldsymbol{q})$ is a constant. We choose the normalization[2]

$$\Phi^{qq}(\boldsymbol{k},\boldsymbol{q}) = 1\,. \tag{3.4}$$

[2]Only the normalization of the full amplitude is relevant in our calculation, so we move all non-trivial factors into the photon-meson impact factor. Caution should therefore be exercised if these impact factors are to be used elsewhere.

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In order to produce the meson, we follow [5] in assuming that the quark-antiquark pair has small transverse size (as determined by the exchanged transverse momentum which is shared approximately equally between the two gluons [1]). This approximation allows us to expand the relevant hadronic matrix element about the light-cone, keeping the leading terms in x^2 (where x is the space-time separation of the quark and antiquark). The relevant light-cone wavefunctions are then taken from the QCD analyses of [13, 14, 15, 5].

The photon wavefunctions in the momentum representation were obtained using the perturbative approach in [16]. These wavefunctions contain spinorial structures with definite chiral parity. The chiral even components (corresponding to vector γ^μ and axial vector $\gamma^\mu\gamma^5$ coupling to quarks) yield non-zero contributions in the massless limit, and the chiral odd ones (tensor $\sigma_{\mu\nu}$) are governed by the quark mass m, and vanish when $m \to 0$. Thus, it is convenient to classify the contributions to the impact factors and amplitudes according to their chiral parity. Then we have

$$\Phi^{\gamma(\lambda)V(\lambda')} = \Phi^{\gamma(\lambda)V(\lambda')}_{\rm even} + \Phi^{\gamma(\lambda)V(\lambda')}_{\rm odd} \tag{3.5}$$

and

$$M_{\lambda\lambda'} = M^{\rm even}_{\lambda\lambda'} + M^{\rm odd}_{\lambda\lambda'}\,, \tag{3.6}$$

for all photon and meson helicities $\lambda = +,-$ and $\lambda' = +,-,0$. In [5] the available information on the vector meson wave functions was employed to derive both the chiral even and the chiral odd components of the amplitudes (3.3). These amplitudes, given at the leading order, are the starting point for our BFKL analysis.

The chiral even impact factors for $\gamma(\lambda)V(\lambda')$ computed in [5] are given by

$$\Phi^{\gamma(+)V(0)}_{\rm even} = -iC_V \int \frac{d^2\boldsymbol{r}\,du}{4\pi}\, m\,K_1(m|\boldsymbol{r}|)\,\frac{\boldsymbol{\epsilon}^{(+)}\cdot\boldsymbol{r}}{|\boldsymbol{r}|}\, f^{\rm dipole}\, f_V\,(1-2u)\,\phi_{||}(u), \tag{3.7}$$

$$\Phi^{\gamma(+)V(+)}_{\rm even} = C_V \int \frac{d^2\boldsymbol{r}\,du}{4\pi}\, m|\boldsymbol{r}|\,K_1(m|\boldsymbol{r}|)\,(\boldsymbol{\epsilon}^{(+)}\cdot\boldsymbol{\epsilon}^{(+)*})\,f^{\rm dipole}\,\times$$
$$\times f_V M_V \frac{u\bar{u}}{2}\left(\int_0^u \frac{dv}{\bar{v}}\,\phi_{||}(v) + \int_u^1 \frac{dv}{v}\,\phi_{||}(v)\right), \tag{3.8}$$

$$\Phi^{\gamma(+)V(-)}_{\rm even} = -C_V \int \frac{d^2\boldsymbol{r}\,du}{4\pi}\, m\,K_1(m|\boldsymbol{r}|)\,\frac{(\boldsymbol{\epsilon}^{(+)}\cdot\boldsymbol{r})(\boldsymbol{\epsilon}^{(-)*}\cdot\boldsymbol{r})}{|\boldsymbol{r}|}\,f^{\rm dipole}\,\times$$
$$\times f_V M_V \left(\bar{u}^2\int_0^u \frac{dv}{\bar{v}}\,\phi_{||}(v) + u^2\int_u^1 \frac{dv}{v}\,\phi_{||}(v)\right), \tag{3.9}$$

with

$$C_V = i\alpha_s^2\frac{N_c^2-1}{N_c^2}eQ_V\,, \tag{3.10}$$

where m is the quark mass, f_V is the meson decay constant, and we take the asymptotic wave function $\phi_{||}(u) = 6\,u(1-u)$ for the light vector mesons. The effective quark charges in the vector mesons are given by $Q_\rho = 1/\sqrt{2}$, $Q_\omega = 1/3\sqrt{2}$, and $Q_\phi = -1/3$.

The chiral odd impact factors for $\gamma(\lambda)V(\lambda')$ with a perturbative (QED) photon wave-function read

$$\begin{aligned}
\Phi_{\text{odd}}^{\gamma(+)V(0)} &= -iC_V \int \frac{d^2\boldsymbol{r}\, du}{4\pi} m\, K_0(m|\boldsymbol{r}|)\, (\boldsymbol{\epsilon}^{(+)} \cdot \boldsymbol{r})\, f^{\text{dipole}} \times \\
&\quad \times f_V^T\, M_V\, u\bar{u} \left(\int_u^1 \frac{dv}{v}\, \phi_\perp(v) - \int_0^u \frac{dv}{\bar{v}}\, \phi_\perp(v) \right), \qquad (3.11)\\
\Phi_{\text{odd}}^{\gamma(+)V(+)} &= C_V \int \frac{d^2\boldsymbol{r}\, du}{4\pi}\, m\, K_0(m|\boldsymbol{r}|)\, (\boldsymbol{\epsilon}^{(+)} \cdot \boldsymbol{\epsilon}^{(+)*})\, f^{\text{dipole}}\, f_V^T \phi_\perp(u)\,, \qquad (3.12)\\
\Phi_{\text{odd}}^{\gamma(+)V(-)} &= -\frac{1}{4} C_V \int \frac{d^2\boldsymbol{r}\, du}{4\pi} m\, K_0(m|\boldsymbol{r}|)\, f^{\text{dipole}} \times \\
&\quad \times (\boldsymbol{\epsilon}^{(+)} \cdot \boldsymbol{r})(\boldsymbol{\epsilon}^{(-)*} \cdot \boldsymbol{r})\, f_V^T\, M_V^2\, 6(u\bar{u})^2\,, \qquad (3.13)
\end{aligned}$$

where $\phi_\perp(u) = 6u\bar{u}$, and the tensor decay constant f_V^T are introduced, and we used the asymptotic form of distribution functions $h_{||}(u)$, $h_3(u)$ and $\phi_\perp(u)$ in (3.13) [5, 13].

In what follows, the complex representation of the two-dimensional transverse vectors will be used, e.g. $r = r_x + ir_y$. This leads to the substitutions $\boldsymbol{r}\boldsymbol{\epsilon}^{(+)} = -r/\sqrt{2}$, $d^2\boldsymbol{r} = d^2r = dr\,dr^*/2i$, $|\boldsymbol{r}| \to |r|$ etc.

Thus, at the lowest order, the chiral even amplitudes for vector meson photoproduction are given by

$$\begin{aligned}
M_{+0}^{\text{even}} &= iC_V \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi}\, m\, \frac{r}{|r|}\, K_1(m|r|)\, f^{\text{dipole}}\, \frac{f_V}{\sqrt{2}}\, (1-2u)\, \phi_{||}(u) \qquad (3.14)\\
M_{++}^{\text{even}} &= C_V \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi}\, m|r|\, K_1(m|r|) f^{\text{dipole}}\, f_V M_V \times \\
&\quad \times \frac{u\bar{u}}{2} \left(\int_0^u \frac{dv}{\bar{v}}\, \phi_{||}(v) + \int_u^1 \frac{dv}{v}\, \phi_{||}(v) \right), \qquad (3.15)\\
M_{+-}^{\text{even}} &= C_V \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi}\, m\, \frac{r^2}{2|r|}\, K_1(m|r|)\, f^{\text{dipole}}\, f_V M_V \times \\
&\quad \times \left(\bar{u}^2 \int_0^u \frac{dv}{\bar{v}}\, \phi_{||}(v) + u^2 \int_u^1 \frac{dv}{v}\, \phi_{||}(v) \right), \qquad (3.16)
\end{aligned}$$

and the chiral odd ones

$$\begin{aligned}
M_{+0}^{\text{odd}} &= i\frac{C_V}{2} \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi} m\, r\, K_0(m|r|)\, f^{\text{dipole}} \times \\
&\quad \times \sqrt{2}\, f_V^T\, M_V u\bar{u} \left(\int_u^1 \frac{dv}{v}\, \phi_\perp(v) - \int_0^u \frac{dv}{\bar{v}}\, \phi_\perp(v) \right), \qquad (3.17)\\
M_{++}^{\text{odd}} &= C_V \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi} m\, K_0(m|r|)\, f^{\text{dipole}}\, f_V^T\, \phi_\perp(u)\,, \qquad (3.18)\\
M_{+-}^{\text{odd}} &= \frac{C_V}{8} \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2r\, du}{4\pi}\, m\, r^2\, K_0(m|r|)\, f^{\text{dipole}}\, f_V^T\, M_V^2\, 6(u\bar{u})^2\,. \qquad (3.19)
\end{aligned}$$

Let us consider the behaviour of the amplitudes in the massless limit. For all chiral even amplitudes, one obtains non-zero results in this limit, as $mK_1(m|\boldsymbol{r}|) \to 1/|\boldsymbol{r}|$

for $m \to 0$. The situation is different for the chiral odd amplitudes. The factor of $mK_0(m|\boldsymbol{r}|) \sim -m\log(m|\boldsymbol{r}|)$ implies the vanishing of all perturbative chiral odd amplitudes in the massless limit. However, as discussed in [5], chiral symmetry breaking effects can induce a sizeable chiral odd coupling of the photon to quarks. In particular, the characteristic mass parameter of the chiral odd coupling was estimated in [5] to be about $2\pi^2 f_\gamma/N_c \simeq 0.46\,\mathrm{GeV}$. Here we account for chiral symmetry breaking by keeping in mind that the quark mass parameter need not be small. We refer to appendix B for a more detailed discussion of the limit $m \to 0$.

We should also remark that we are able to reproduce the original formulae of [16] by considering the only surviving amplitudes in the limit that the quark and anti-quark are collinear, i.e. $M^{\rm odd}_{++}$ and $M^{\rm even}_{+0}$. We also note that if we make the further constraint that the quark and anti-quark share the meson's momentum equally, we obtain the formulae of [1] for the M_{++} amplitude, the only surviving amplitude in this case. In [10], this amplitude was shown to be in good agreement with the ZEUS data (after including BFKL effects) allowing us to conclude that a sizeable chiral odd contribution seems to be required by the data.

4. The exact BFKL amplitude

4.1 Generalities

The BFKL kernel in the leading logarithmic approximation exhibits, in the impact parameter representation, invariance under conformal transformations [17]. The conformal symmetry of the kernel permits the following expansion of the amplitude in the basis of eigenfunctions $E_{n,\nu}$ [17]:

$$M_{AB}(z,q) = \frac{1}{(2\pi)^2}\sum_{n=-\infty}^{n=\infty}\int_{-\infty}^{\infty} d\nu\,\frac{\nu^2+n^2/4}{[\nu^2+(n-1)^2/4][\nu^2+(n+1)^2/4]}\times \\ \times \exp[\chi_n(\nu)z]\, I^A_{n,\nu}(q)\,(I^B_{n,\nu}(q))^* \tag{4.1}$$

where

$$\chi_n(\nu) = 4\mathcal{R}\mathrm{e}\left(\psi(1)-\psi\left(\frac{1}{2}+\frac{|n|}{2}+i\nu\right)\right) \tag{4.2}$$

is proportional to the eigenvalues of the BFKL kernel and

$$z = \frac{3\alpha_s}{2\pi}\ln\left(\frac{\hat{s}}{\Lambda^2}\right) \tag{4.3}$$

(Λ is a characteristic mass scale related to M_V^2 and $|t|$).

$$I^A_{n,\nu}(q) = \int\frac{d^2k}{(2\pi)^2}\Phi^A(k,q)\int d^2\rho_1\,d^2\rho_2\,E_{n,\nu}(\rho_1,\rho_2)\exp(ik\cdot\rho_1+i(q-k)\cdot\rho_2)\,, \tag{4.4}$$

and analogously for the index B. The eigenfunctions are given by

$$E_{n,\nu}(\rho_1,\rho_2) = \left(\frac{\rho_1-\rho_2}{\rho_1\rho_2}\right)^{-\tilde{\mu}+1/2}\left(\left(\frac{\rho_1-\rho_2}{\rho_1\rho_2}\right)^*\right)^{-\mu+1/2} \tag{4.5}$$

where $\mu = n/2 - i\nu$ and $\tilde{\mu} = -n/2 - i\nu$. Here k and q are transverse two dimensional momentum vectors, and ρ_1 and ρ_2 are position space vectors in the standard complex representation. The functions $\Phi^A = \Phi^{\gamma(\lambda)V(\lambda')}$ and $\Phi^B = \Phi^{qq}$ are the impact factors.

The quark impact factor in representation (4.4) was found in [18], generalizing the Mueller-Tang subtraction [19] to non-zero conformal spin;

$$I^{qq}_{n,\nu}(q) = -\frac{4\pi\, i^n}{|q|}\left(\frac{|q|^2}{4}\right)^{i\nu}\left(\frac{q^*}{q}\right)^{n/2}\frac{\Gamma(1/2+n/2-i\nu)}{\Gamma(1/2+n/2+i\nu)} \qquad (4.6)$$

for even n and $I^{qq}_{n,\nu} = 0$ for odd n.

We now compute the impact factors for the $A = \gamma(\lambda)V(\lambda')$ transition for all helicity states and all conformal spins n. The following integrals are to be evaluated

$$I^A_{n,\nu}(q) = \int\frac{d^2k}{(2\pi)^2}\,\Phi^A(k,q)\int d^2\rho_1\, d^2\rho_2\, E_{n,\nu}(\rho_1,\rho_2)\times$$
$$\times\exp\left(\frac{ik^*\rho_1}{2}+\frac{ik\rho_1^*}{2}+\frac{i(q^*-k^*)\rho_2}{2}+\frac{i(q-k)\rho_2^*}{2}\right). \qquad (4.7)$$

Changing variables to $\rho_1 = R+\rho/2$, $\rho_2 = R-\rho/2$ and integrating over d^2k we get

$$I^A_{n,\nu}(q) = \int d^2\rho\,\Phi^A_q(\rho)\int d^2R\, E_{n,\nu}(R,\rho)e^{iq\cdot R}, \qquad (4.8)$$

where $\Phi^A_q(\rho)$ is the Fourier transform of the impact factor in the momentum space $\Phi^A(\boldsymbol{k},\boldsymbol{q})$,

$$\Phi^A_q(\rho) = \int\frac{d^2k}{(2\pi)^2}\Phi^A(k,q)e^{i(k-q/2)\cdot\rho}. \qquad (4.9)$$

The integration over R in (4.8) has been done by Navelet and Peschanski [20], resulting in

$$I^A_{n,\nu}(q) = \int d^2\rho\,\Phi^A_q(\rho)\,\hat{E}^q_{n,\nu}(\rho), \qquad (4.10)$$

where the mixed representation eigenfunction $E^q_{n,\nu}(\rho)$ is given by

$$\hat{E}^q_{n,\nu} = \frac{(-1)^n}{2\pi^2}\,b_{n,\nu}\left(\frac{|q|}{8}\right)^{2i\nu}\left(\frac{q^*}{q}\right)^{n/2}\Gamma\left(1-i\nu-\frac{n}{2}\right)\Gamma\left(1-i\nu+\frac{n}{2}\right)\times$$
$$\times|\rho|\left[J_\mu\left(\frac{q^*\rho}{4}\right)J_{\tilde{\mu}}\left(\frac{q\rho^*}{4}\right)-(-1)^n J_{-\mu}\left(\frac{q^*\rho}{4}\right)J_{-\tilde{\mu}}\left(\frac{q\rho^*}{4}\right)\right] \qquad (4.11)$$

and

$$b_{n,\nu} = \frac{2^{4i\nu}\pi^3}{|n/2|-i\nu}\,\frac{\Gamma(|n|/2-i\nu+1/2)\Gamma(|n|/2+i\nu)}{\Gamma(|n|/2+i\nu+1/2)\Gamma(|n|/2-i\nu)}. \qquad (4.12)$$

The integral over k of impact factors (3.8), (3.7), (3.9) and (3.12), (3.11), (3.13) may be easily performed since k appears only in $f^{\rm dipole}$. Due to the resulting δ functions $\delta(\rho)$, $\delta(r-\rho)$ and $\delta(r+\rho)$ the integration over r is trivial. Using the fact that $\hat{E}^q_{n,\nu}(\rho) = 0$ for $\rho = 0$, and

$$\hat{E}^q_{n,\nu}(\rho) = (-1)^n\hat{E}^q_{n,\nu}(-\rho) \qquad (4.13)$$

one arrives, for all impact factors, at complex integrals of the type

$$I_{\alpha\beta}(\nu, n, q, u; a) = m \int d^2\rho\, \rho^{\alpha+1} \rho^{*\,\beta+1}\, K_a(m|\rho|) e^{\frac{i\xi}{4}[q^*\rho + q\rho^*]} \times \\ \times \left[J_\mu\left(\frac{q^*\rho}{4}\right) J_{\tilde\mu}\left(\frac{q\rho^*}{4}\right) - (-1)^n J_{-\mu}\left(\frac{q^*\rho}{4}\right) J_{-\tilde\mu}\left(\frac{q\rho^*}{4}\right)\right] \tag{4.14}$$

where $\xi = 2u - 1$ and $K_a(x)$ is the modified Bessel function, displaying effects of the quark mass. The parameter a equals 1 for the chiral-even and 0 for the chiral-odd impact factors of [5].

4.2 Amplitudes

The helicity amplitudes can then be expressed in the following way

$$M^{\rm even}_{+0} = \frac{iC_V f_V}{4\sqrt{2}|q|} \int_0^1 du\, 6\, u(1-u)\,(1-2u) \times \tag{4.15}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{0-1}(\nu, 2n, q, u; 1)\,,$$

$$M^{\rm even}_{++} = \frac{C_V f_V M_V}{8|q|} \int_0^1 du\, 6\, u(1-u)\,(u^2-u+1/2) \times \tag{4.16}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{00}(\nu, 2n, q, u; 1)\,,$$

$$M^{\rm even}_{+-} = \frac{C_V f_V M_V}{8|q|} \int_0^1 du\, 6\, u^2\,(1-u)^2 \times \tag{4.17}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{+1-1}(\nu, 2n, q, u; 1)\,,$$

where we performed integration over v in (3.14), (3.15), (3.15).

Analogously, one may express the chiral odd amplitudes

$$M^{\rm odd}_{+0} = \frac{iC_V f_V^T M_V}{8\sqrt{2}|q|} \int_0^1 du\, 6\, u(1-u)\,(1-2u) \times \tag{4.18}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{\frac12-\frac12}(\nu, 2n, q, u; 0)\,,$$

$$M^{\rm odd}_{++} = \frac{C_V f_V^T}{4|q|} \int_0^1 du\, 6\, u(1-u) \times \tag{4.19}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{-\frac12-\frac12}(\nu, 2n, q, u; 0)$$

$$M^{\rm odd}_{+-} = \frac{C_V f_V^T M_V^2}{32|q|} \int_0^1 du\, 6\, u^2\,(1-u)^2 \times \tag{4.20}$$
$$\times \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu)z]}{\sin(i\pi\nu)} I_{\frac32-\frac12}(\nu, 2n, q, u; 0)\,.$$

The derivation of integrals (4.14) for integer difference[3] $\alpha-\beta$ is given in appendix A, and the result reads

$$
\begin{aligned}
I_{\alpha\beta}(\nu,n,q,u;a) = \frac{m}{2}\int_{C'-i\infty}^{C'+i\infty}\frac{d\zeta}{2\pi i}\Gamma\left(\frac{a}{2}-\zeta\right)\Gamma\left(-\frac{a}{2}-\zeta\right)\tau_q^{\zeta}\left(i\,\mathrm{sign}\,(1-2u)\right)^{\alpha-\beta+n}\times \\
\times\left(\frac{4}{|q|}\right)^4\Big[\sin\pi(\alpha+\mu+\zeta)B(\alpha,\mu,q^*,u,\zeta)B(\beta,\widetilde{\mu},q,u^*,\zeta)- \\
-(-1)^n\sin\pi(\alpha-\mu+\zeta)B(\alpha,-\mu,q^*,u,\zeta)B(\beta,-\widetilde{\mu},q,u^*,\zeta)\Big]
\end{aligned}
\tag{4.21}
$$

where we have introduced the dimensionless parameter $\tau_q = 4m^2/|q|^2$ and the conformal blocks

$$
\begin{aligned}
B(\alpha,\mu,q^*,u,\zeta) = (-4u\bar{u})^{-(\mu+2+\alpha+\zeta)/2}\left(\frac{4}{q^*}\right)^{\alpha}2^{-\mu}\,\frac{\Gamma(\mu+2+\alpha+\zeta)}{\Gamma(\mu+1)}\times \\
\times\,{}_2F_1\left(\frac{\mu+2+\alpha+\zeta}{2},\,\frac{\mu-1-\alpha-\zeta}{2}\,;\,\mu+1\,;\,\frac{1}{4u\bar{u}}\right).
\end{aligned}
\tag{4.22}
$$

Note, that the sums are performed over even conformal spins $2n$ due to properties of the quark impact factor (4.6). It turns out that odd conformal spin components of photon-meson impact factors (3.7), (3.8), (3.9) also vanish. The impact factors are symmetric under the coordinate transformation given by $(\rho\to-\rho,\quad u\to 1-u)$. The parity behaviour (4.13) of conformal eigenfunctions applied to (4.21) leads to the relation

$$
I_{\alpha\beta}(\nu,n,q,u;a) = (-1)^{n+\alpha-\beta}I_{\alpha\beta}(\nu,n,q,1-u;a)\,. \tag{4.23}
$$

These results may be combined to give $I_{n,\nu}^{\gamma(\lambda)V(\lambda')}=0$ for odd n.

The obtained expressions for the impact factors are rather complicated and the derivation is rather involved. Therefore we performed an independent test of the results by comparing a direct numerical integration of (4.14) over ρ to the analytical formula (4.21) for some arbitrary sets of parameters, with the quark mass tending to zero. Full agreement was found in all cases.

5. Conclusions and outlook

Helicity amplitudes for diffractive vector meson photoproduction off a proton were investigated using perturbative QCD methods, focusing on scattering occurring at high energies and high momentum transfer, for which the target proton dissociates. The process is described in terms of hard colour singlet exchange between a parton from the proton and the quark-antiquark component of the photon.

We re-derived all the relevant impact factors in terms of the meson light-cone wavefunction. The helicity amplitudes were given in the two gluon exchange approximation with arbitrary quark mass and classified according to their chiral parity. We suggest that the perturbative formulae with the constituent quark mass reasonably approximates the effects of chiral symmetry breaking.

[3]The condition of integer $\alpha-\beta$ is necessary for the amplitudes to be single valued.

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As the scattering occurs in Regge kinematics, the perturbative corrections are enhanced by large logarithms of the energy, and a full resummation of the leading logarithmic corrections is necessary. Therefore, QCD corrections to the lowest order amplitudes were also considered. The amplitudes of the colour singlet exchange may then be described in terms of the BFKL equation. Thus, we derived the projections of the photon-meson impact factors on the conformal eigenstates of the BFKL kernel for an arbitrary conformal spin. Both the chiral even and chiral odd impact factors are obtained in terms of a single complex line integral. Using those results, the BFKL evolved helicity amplitudes describing the meson photoproduction off a parton were given in an analytical form. These are the main results of this paper.

Our results are correct in the leading logarithmic approximation (LLA). Consequently, we are unable to make reliable predictions for the absolute normalization of the cross-sections (due to the ambiguity in the scale Λ). Moreover, next-to-leading logarithms [21, 22] are well known to be important in the case of the $t = 0$ BFKL equation (they have yet to be investigated in the non-forward case). Attempts to incorporate at least some of the next-to-leading logarithms have been performed and indicate that the LLA is probably quite reasonable provided one treats the leading eigenvalue of the kernel as an effective parameter whose value in LLA is determined by fixing α_s [11, 23]. The fixing of α_s is also supported by the work of [24, 25].

It remains to confront our calculations with the t-dependence of the cross-section and the spin density matrix elements which have been measured at HERA [3, 4] for the ρ, ϕ and J/ψ mesons. This study will be the subject of a second paper [7], and should provide a stringent test of the BFKL approach, in addition to constraining the photon and meson wavefunctions.

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A. The integral for arbitrary quark mass

It is convenient to use the variable $\xi = u - \bar{u} = 2u - 1$ rather than u, and to compute the integral for ξ an arbitrary complex number and then continue analytically into the physical region $-1 < \xi < 1$. We can then write (4.14), using $2i\, d^2\rho = d\rho\, d\rho^*$, as

$$\mathcal{I}_\pm = m \int \frac{d\rho\, d\rho^*}{2i}\, \rho^{\alpha+1} \rho^{*\,\beta+1}\, K_a(m|\rho|)\, e^{\frac{i}{4}(\xi^* \rho^* q + \xi \rho q^*)}\, J_{\pm\mu}\left(\frac{q^*\rho}{4}\right) J_{\pm\tilde{\mu}}\left(\frac{q\rho^*}{4}\right) \qquad \text{(A.1)}$$

so that $\mathcal{I} = \mathcal{I}_+ - (-1)^n\, \mathcal{I}_-$.

Let us start by computing the integral (A.1) for massive quarks. The massless case will be obtained in a suitable limit.

The Bessel functions J are defined with a cut, but the products appearing here are single-valued. To make the cut structure of the integral more explicit it is convenient to represent the Bessel functions in terms of confluent hypergeometric functions, which are individually single-valued, by using the relation [26]

$$J_{\sigma-1/2}(z) = \phi(\sigma, 2\sigma; -2iz) \left(\frac{z}{2}\right)^{\sigma-1/2} \frac{e^{iz}}{\Gamma(\sigma+1/2)}\,. \tag{A.2}$$

Inserting this into eq. (A.1) we have

$$\begin{aligned}\mathcal{I}_+ =& \frac{64^{i\nu}\, q^{*\,\mu} q^{\widetilde{\mu}}}{\Gamma(\mu+1)\Gamma(\widetilde{\mu}+1)}\, m \int \frac{d\rho\, d\rho^*}{2i} \rho^{\alpha+1+\mu} \rho^{*\,\beta+1+\widetilde{\mu}} \times \\ &\times \exp\left[i\,\frac{\xi+1}{4} q^*\rho + i\,\frac{\xi^*+1}{4} q\rho^*\right] K_a(m|\rho|) \times \\ &\times \phi\left(\mu+\frac{1}{2}, 2\mu+1; -\frac{i}{2}q^*\rho\right) \phi\left(\widetilde{\mu}+\frac{1}{2}, 2\widetilde{\mu}+1; -\frac{i}{2}q\rho^*\right).\end{aligned} \tag{A.3}$$

The Bessel function K_a can be replaced with the line integral representation[4]

$$K_a(x) = \frac{1}{2}\left(\frac{2}{x}\right)^b \int_{C-i\infty}^{C+i\infty} \frac{dz}{2\pi i} \Gamma\left(\frac{b}{2}+\frac{a}{2}-z\right) \Gamma\left(\frac{b}{2}-\frac{a}{2}-z\right) \left(\frac{x^2}{4}\right)^z \tag{A.4}$$

for $C \leq Re[b/2+a/2]$, $Re[b/2-a/2]$. Substituted into eq. (A.3), we now get the following integral:

$$\begin{aligned}\mathcal{I}_+ =& \frac{64^{i\nu}\, q^{*\,\mu} q^{\widetilde{\mu}}}{\Gamma(\mu+1)\Gamma(\widetilde{\mu}+1)} \frac{m}{2} \int_{C'-i\infty}^{C'+i\infty} \frac{d\zeta}{2\pi i} \Gamma(a/2-\zeta)\Gamma(-a/2-\zeta) \left(\frac{m^2}{4}\right)^\zeta \times \\ &\times \int \frac{d\rho\, d\rho^*}{2i} \rho^{\alpha+1+\mu+\zeta} \rho^{*\,\beta+1+\widetilde{\mu}+\zeta} \exp\left[i\,\frac{\xi+1}{4} q^*\rho + i\,\frac{\xi^*+1}{4} q\rho^*\right] \times \\ &\times \phi\left(\mu+\frac{1}{2}, 2\mu+1; -\frac{i}{2}q^*\rho\right) \phi\left(\widetilde{\mu}+\frac{1}{2}, 2\widetilde{\mu}+1; -\frac{i}{2}q\rho^*\right)\end{aligned} \tag{A.5}$$

where we have changed to the integration variable $\zeta = z - b/2$.

The ρ and ρ^* dependence of the integrand is clearly factorized, but to disentangle the integrals we have to consider carefully the analytic structure. It is convenient to rescale the integration variables,

$$W = \frac{q^*(\xi+1)}{4i}\rho \tag{A.6}$$

$$Z = \frac{q(\xi^*+1)}{4i}\rho^*\,, \tag{A.7}$$

[4] This can be seen either by the inverse Mellin transformation or by expressing K_a as a Meijer G function and using its line integral representation.

giving

$$\mathcal{I}_+ = \frac{64^{i\nu}\, q^{*\,\mu} q^{\widetilde{\mu}}}{\Gamma(\mu+1)\Gamma(\widetilde{\mu}+1)} \frac{m}{2} \int_{C'-i\infty}^{C'+i\infty} \frac{d\zeta}{2\pi i} \Gamma(a/2-\zeta)\Gamma(-a/2-\zeta) \left(\frac{m^2}{4}\right)^{\zeta} \times$$
$$\times \int \frac{dW\, dZ}{2i} W^{\alpha+1+\mu+\zeta}\, Z^{\beta+1+\widetilde{\mu}+\zeta}\, e^{-(W+Z)} \times$$
$$\times \left(\frac{4i}{q^*(\xi+1)}\right)^{2+\alpha+\mu+\zeta} \left(\frac{4i}{q(\xi^*+1)}\right)^{2+\beta+\widetilde{\mu}+\zeta} \times$$
$$\times \phi\left(\mu+\frac{1}{2}, 2\mu+1; \frac{2W}{\xi+1}\right) \phi\left(\widetilde{\mu}+\frac{1}{2}, 2\widetilde{\mu}+1; \frac{2Z}{\xi^*+1}\right). \tag{A.8}$$

Let us now fix Z to be a constant real number. If $Z > 0$ there is a branch cut along the positive real axis from zero to infinity in the complex W plane, and if $Z < 0$ the cut is along the negative real axis. For the integral to converge Z must be positive, and so to get a non-zero contribution we have to close the contour around the cut in the right half-plane, finding the discontinuity across the cut. Keeping Z fixed, the W integral can be written as

$$\oint_C dW\, (-WZ)^u e^{-W} e^{-Z} F(W) \tag{A.9}$$

where u is a non-integer complex exponent and $F(W)$ is the non-problematic, single-valued part of the integrand. Let the contour C be the two rays $W_\pm = re^{\pm i\delta}$, where r is integrated from 0 to infinity and δ is a positive real number, anticipating the limit $\delta \to 0$. We then have

$$\oint_C dW(-WZ)^u e^{-W} e^{-Z} F(W) = \lim_{\delta\to 0} \int_0^\infty dr[(-ZW_-)^u e^{-W_-} - (-ZW_+)^u e^{-W_+}] e^{-Z} F(W)$$
$$= 2i \sin(\pi u)\, Z^u e^{-Z} \int_0^\infty dr\, r^u e^{-r} F(r)\,. \tag{A.10}$$

That is, factorizing the two integrals, we get an extra factor $\sin(\pi u)$ ensuring that the expression is single-valued. We have chosen the direction of the contour that gives the correct overall sign. Applying this to (A.8), with $u = \alpha + \mu + \zeta$, and using the fact that $\mu - \widetilde{\mu} = n$ and $\alpha - \beta$ are both integers, leads to

$$\mathcal{I}_+ = \frac{64^{i\nu}\, q^{*\,\mu} q^{\widetilde{\mu}}}{\Gamma(\mu+1)\Gamma(\widetilde{\mu}+1)} \frac{m}{2} \int_{C'-i\infty}^{C'+i\infty} \frac{d\zeta}{2\pi i} \Gamma(a/2-\zeta)\Gamma(-a/2-\zeta) \left(\frac{m^2}{4}\right)^{\zeta} \times$$
$$\times \left(\frac{4}{q^*(\xi+1)}\right)^{2+\alpha+\mu+\zeta} \left(\frac{4}{q(\xi^*+1)}\right)^{2+\beta+\widetilde{\mu}+\zeta} \sin\pi(\alpha+\mu+\zeta)\,(-i)^{\alpha-\beta+n} \times$$
$$\times \int_0^\infty dW\, W^{\alpha+1+\mu+\zeta}\, e^{-W} \phi\left(\mu+\frac{1}{2}, 2\mu+1; \frac{2W}{\xi+1}\right) \times$$
$$\times \int_0^\infty dZ\, Z^{\beta+1+\widetilde{\mu}+\zeta}\, e^{-Z} \phi\left(\widetilde{\mu}+\frac{1}{2}, 2\widetilde{\mu}+1; \frac{2Z}{\xi^*+1}\right). \tag{A.11}$$

The last two integrals are instances of the standard integral [26]

$$\int_0^\infty du\, e^{-u} u^b \phi(a, 2a, ku) = \Gamma(b+1)\, {}_2F_1(a, b+1; 2a; k)\,, \tag{A.12}$$

and when using this, (A.11) is reduced to the single line integral

$$\mathcal{I}_{+} = \frac{64^{i\nu}\, q^{*\,\mu} q^{\widetilde{\mu}}}{\Gamma(\mu+1)\Gamma(\widetilde{\mu}+1)}\,\frac{m}{2}\int_{C'-i\infty}^{C'+i\infty}\frac{d\zeta}{2\pi i}\Gamma(a/2-\zeta)\Gamma(-a/2-\zeta)\left(\frac{m^2}{4}\right)^{\zeta}\times$$
$$\times\left(\frac{4}{q^*(\xi+1)}\right)^{2+\alpha+\mu+\zeta}\left(\frac{4}{q(\xi^*+1)}\right)^{2+\beta+\widetilde{\mu}+\zeta}\sin\pi(\alpha+\mu+\zeta)\,(-i)^{\alpha-\beta+n}\times$$
$$\times\Gamma(\alpha+2+\mu+\zeta)\ {}_2F_1\left(\mu+\frac{1}{2},\,\alpha+2+\mu+\zeta;\,2\mu+1;\,\frac{2}{\xi+1}\right)\times$$
$$\times\Gamma(\beta+2+\widetilde{\mu}+\zeta)\ {}_2F_1\left(\widetilde{\mu}+\frac{1}{2},\,\beta+2+\widetilde{\mu}+\zeta;\,2\widetilde{\mu}+1;\,\frac{2}{\xi^*+1}\right). \tag{A.13}$$

We are now ready to express $\mathcal{I}$ in its final form for n even,

$$\begin{aligned}\mathcal{I} &= \mathcal{I}_{+} - (-1)^n\,\mathcal{I}_{-}\\ &= \frac{m}{2}\int_{C'-i\infty}^{C'+i\infty}\frac{d\zeta}{2\pi i}\Gamma(a/2-\zeta)\Gamma(-a/2-\zeta)\,\tau_q^{\zeta}\,\left(\frac{4}{|q|}\right)^4(-i)^{\alpha-\beta+n}\times\\ &\quad\times\Big[\sin\pi(\alpha+\mu+\zeta)B'(\alpha,\mu,q^*,\xi,\zeta)\,B'(\beta,\widetilde{\mu},q,\xi^*,\zeta)-\\ &\qquad -\sin\pi(\alpha-\mu+\zeta)\,B'(\alpha,-\mu,q^*,\xi,\zeta)\,B'(\beta,-\widetilde{\mu},q,\xi^*,\zeta)\Big]\,,\end{aligned} \tag{A.14}$$

where we have introduced the dimensionless parameter $\tau_q = 4m^2/|q|^2$ and the conformal blocks

$$\begin{aligned}B'(\alpha,\mu,q^*,\xi,\zeta) &= \left(\frac{1}{\xi+1}\right)^{\mu+2+\alpha+\zeta}\left(\frac{4}{q^*}\right)^{\alpha}2^{-\mu}\,\frac{\Gamma(\alpha+2+\mu+\zeta)}{\Gamma(\mu+1)}\times\\ &\quad\times{}_2F_1\left(\mu+\frac{1}{2}\,,\,\mu+2+\alpha+\zeta\,;\,2\mu+1\,;\,\frac{2}{\xi+1}\right).\end{aligned} \tag{A.15}$$

We can make the replacement

$$B'(\alpha,\mu,q^*,\xi,\zeta)\to B''(\alpha,\mu,q^*,\xi,\zeta), \tag{A.16}$$

where

$$\begin{aligned}B''(\alpha,\mu,q^*,\xi,\zeta) &= \left(\frac{|\xi|}{\xi}\right)^{\alpha+\mu}(\xi^2-1)^{-(\mu+2+\alpha+\zeta)/2}\left(\frac{4}{q^*}\right)^{\alpha}2^{-\mu}\,\frac{\Gamma(\mu+2+\alpha+\zeta)}{\Gamma(\mu+1)}\times\\ &\quad\times{}_2F_1\left(\frac{\mu+2+\alpha+\zeta}{2}\,,\,\frac{\mu-1-\alpha-\zeta}{2}\,;\,\mu+1\,;\,\frac{1}{1-\xi^2}\right),\end{aligned} \tag{A.17}$$

by using the identities 9.134:1 and 9.131:1 of [26] and assuming n is even, considering that the conformal block will be multiplied with another one. This displays explicitly that the $\xi\leftrightarrow-\xi$ symmetry depends on α and β; there will be an overall factor in the product of conformal blocks,

$$\left(\frac{|\xi|}{\xi}\right)^{\alpha+\mu}\left(\frac{|\xi^*|}{\xi^*}\right)^{\beta+\widetilde{\mu}} = (\text{sign}\ \xi)^{\alpha-\beta+n}\,, \tag{A.18}$$

when analytically continuing into the physical region $-1<\xi<1$. This factor is pulled out of the conformal blocks in equation (4.22), appearing in the main body of the text.

The results (A.14) and (A.17) show features of our earlier calculations of heavy [11] vector meson photoproduction for $a = 0$. For $m \to 0$, it gives the amplitudes for light vector meson production in the massless case [27]. In appendix B we will show the result in the massless quark limit, and how to obtain it from the present massive quark result.

Note that when evaluating these expressions numerically it is necessary to carefully consider the branch cut structure of the conformal blocks, which are not single-valued individually. It is then useful to further transform eqs. (A.15) or (A.17) by e.g. using formula 9.132:2 of [26], isolating the branch cuts outside of the hypergeometric functions.

B. The massless quark limit

In [27] the formulae corresponding to (A.14) and (A.17) were computed in the approximation of zero quark mass. This corresponds to using the small argument behaviour for the case $a = 1$, relevant for the calculation of chiral even amplitudes,

$$mK_1(m|\rho|) \sim \frac{1}{|\rho|}\,. \tag{B.1}$$

Then, one obtains a simpler result $\mathcal{J} = \mathcal{I}|_{m=0}$ not involving the line integral over ζ:

$$\begin{aligned}\mathcal{J} = \left(\frac{4}{|q|}\right)^4 (-i)^{\alpha-\beta+n}\Big[&\sin\pi\left(\alpha+\mu-\frac{1}{2}\right) C(\alpha,\mu,q^*,\xi)\, C(\beta,\widetilde{\mu},q,\xi^*) - \\ &-(-1)^n \sin\pi\left(\alpha-\mu-\frac{1}{2}\right) C(\alpha,-\mu,q^*,\xi)\, C(\beta,-\widetilde{\mu},q,\xi^*)\Big]\,,\end{aligned} \tag{B.2}$$

where the $C(\alpha,\mu,q^*,\xi)$ conformal blocks are given by[5] $B''(\alpha,\mu,q^*,\xi,\zeta=-1/2)$

$$\begin{aligned}C(\alpha,\mu,q^*,\xi) = &\left(\frac{|\xi|}{\xi}\right)^{\alpha+\mu} (\xi^2-1)^{-(\mu+3/2+\alpha)/2} \left(\frac{4}{q^*}\right)^{\alpha} 2^{-\mu}\,\frac{\Gamma(\mu+3/2+\alpha)}{\Gamma(\mu+1)} \times \\ &\times {}_2F_1\left(\frac{\mu+3/2+\alpha}{2}\,, \frac{\mu-1/2-\alpha}{2}\,; \mu+1\,; \frac{1}{1-\xi^2}\right).\end{aligned} \tag{B.3}$$

The calculation proceeds in a similar way to the preceding one and we do not show the details here. It should, however, be possible to recover the massless result from the massive one by taking the limit $m \to 0$. This can be done by taking the leading pole approximation for the ζ integral in the limit $\tau_q = 4m^2/|q|^2 \to 0$. The product of Gamma functions in the integrand becomes $\Gamma(1/2-\zeta)\Gamma(-1/2-\zeta)$ and there is a simple pole at $\zeta = -1/2$ and double poles at $\zeta = 1/2, 3/2, \ldots$. The double poles, however, vanish in the limit $\tau_q \to 0$, and using the residue theorem for the pole at $\zeta = -1/2$ reproduces the massless result (B.2). Note that formula (B.2) is a generalization of an integral calculated in [20] for $\alpha = \beta$, and we have checked that our result agrees.

The perturbative chiral-odd photon wave function corresponds to $a = 0$ and is proportional to $mK_0(m|\rho|) \to -m\ln(m|\rho|)$, thus it vanishes in the massless limit.

[5]The substitution $\zeta = -1/2$ is made to keep the definitions of α and β the same, because there is a difference by a factor of $|\rho|$ between the massless and massive integrand.

We stress that the above formulae (i.e. in the massless limit) are only reliable away from the regions of u close to 0 or 1. This is because the substitution B.1 breaks down as $\rho \to \infty$. In effect the expansion parameter is more like $m^2/(u(1-u)|q|^2)$ and this is not small at the endpoints. In particular, a careful treatment reveals that there are no endpoint singularities in the full amplitude in the massless limit, for any individual conformal spin. In the $z \to 0$ (two-gluon) limit, the endpoint singularities shown by [5] in the $M_{++}^{\rm even}$ amplitude are recovered by a non-convergent sum over all conformal spins, analogous to that demonstrated in [18] for $qq \to qq$ scattering.

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